OPERACIONES DIFERENCIALES VECTORIALES EN COORDENADAS ORTOGONALES

B
APENDICE

B1. Coordenadas rectangulares

$$\nabla\psi = \frac{\partial\psi}{\partial x}\mathbf{i} + \frac{\partial\psi}{\partial y}\mathbf{j} + \frac{\partial\psi}{\partial z}\mathbf{k}, \qquad \nabla\cdot\mathbf{f} = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}, \qquad \nabla^2\psi = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2},$$

$$\nabla\times\mathbf{f} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ f_1 & f_2 & f_3 \end{vmatrix} = \left(\frac{\partial f_3}{\partial y} - \frac{\partial f_2}{\partial z}\right)\mathbf{i} + \left(\frac{\partial f_1}{\partial z} - \frac{\partial f_3}{\partial x}\right)\mathbf{j} + \left(\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y}\right)\mathbf{k}.$$

B2. Coordenadas cilíndricas

$$\nabla\psi = \frac{\partial\psi}{\partial\rho}\mathbf{e}_\rho + \frac{1}{\rho}\frac{\partial\psi}{\partial\phi}\mathbf{e}_\phi + \frac{\partial\psi}{\partial z}\mathbf{k}, \qquad \nabla\cdot\mathbf{f} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho f_\rho) + \frac{1}{\rho}\frac{\partial f_\phi}{\partial\phi} + \frac{\partial f_3}{\partial z}, \qquad \nabla^2\psi = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\psi}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2\psi}{\partial\phi^2} + \frac{\partial^2\psi}{\partial z^2},$$

$$\nabla\times\mathbf{f} = \frac{1}{\rho}\begin{vmatrix} \mathbf{e}_\rho & \rho\,\mathbf{e}_\phi & \mathbf{k} \\ \dfrac{\partial}{\partial\rho} & \dfrac{\partial}{\partial\phi} & \dfrac{\partial}{\partial z} \\ f_\rho & f_\phi & f_3 \end{vmatrix} = \frac{1}{\rho}\left(\frac{\partial f_3}{\partial\phi} - \frac{\partial f_\phi}{\partial z}\right)\mathbf{e}_\rho + \left(\frac{\partial f_\rho}{\partial z} - \frac{\partial f_3}{\partial\rho}\right)\mathbf{e}_\phi + \frac{1}{\rho}\left(\frac{\partial f_\phi}{\partial\rho} - \frac{\partial f_\rho}{\partial\phi}\right)\mathbf{k}.$$

B3. Coordenadas esféricas

$$\nabla\psi = \frac{\partial\psi}{\partial r}\mathbf{e}_r + \frac{1}{r}\frac{\partial\psi}{\partial\theta}\mathbf{e}_\theta + \frac{1}{r\,\text{sen}\,\theta}\frac{\partial\psi}{\partial\phi}\mathbf{e}_\phi,$$

$$\nabla\cdot\mathbf{f} = \frac{1}{r^2\,\text{sen}\,\theta}\left[\text{sen}\,\theta\frac{\partial}{\partial r}(r^2 f_r) + r\frac{\partial}{\partial\theta}(\text{sen}\,\theta\, f_\theta) + r\frac{\partial f_\phi}{\partial\phi}\right] = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 f_r) + \frac{1}{r\,\text{sen}\,\theta}\frac{\partial}{\partial\theta}(\text{sen}\,\theta\, f_\theta) + \frac{1}{r\,\text{sen}\,\theta}\frac{\partial f_\phi}{\partial\phi},$$

$$\begin{aligned}\nabla^2\psi &= \frac{1}{r^2\,\text{sen}\,\theta}\left[\text{sen}\,\theta\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{\partial}{\partial\theta}\left(\text{sen}\,\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{\text{sen}\,\theta}\frac{\partial^2\psi}{\partial\phi^2}\right] \\ &= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\,\text{sen}\,\theta}\frac{\partial}{\partial\theta}\left(\text{sen}\,\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\,\text{sen}^2\,\theta}\frac{\partial^2\psi}{\partial\phi^2}.\end{aligned}$$